

C4

DIFFERENTIATION

Worksheet C

- 1 Differentiate with respect to x
- a** $4y$ **b** y^3 **c** $\sin 2y$ **d** $3e^{y^2}$
- 2 Find $\frac{dy}{dx}$ in terms of x and y in each case.
- a** $x^2 + y^2 = 2$ **b** $2x - y + y^2 = 0$ **c** $y^4 = x^2 - 6x + 2$
d $x^2 + y^2 + 3x - 4y = 9$ **e** $x^2 - 2y^2 + x + 3y - 4 = 0$ **f** $\sin x + \cos y = 0$
g $2e^{3x} + e^{-2y} + 7 = 0$ **h** $\tan x + \operatorname{cosec} 2y = 1$ **i** $\ln(x - 2) = \ln(2y + 1)$
- 3 Differentiate with respect to x
- a** xy **b** x^2y^3 **c** $\sin x \tan y$ **d** $(x - 2y)^3$
- 4 Find $\frac{dy}{dx}$ in terms of x and y in each case.
- a** $x^2y = 2$ **b** $x^2 + 3xy - y^2 = 0$ **c** $4x^2 - 2xy + 3y^2 = 8$
d $\cos 2x \sec 3y + 1 = 0$ **e** $y = (x + y)^2$ **f** $xe^y - y = 5$
g $2xy^2 - x^3y = 0$ **h** $y^2 + x \ln y = 3$ **i** $x \sin y + x^2 \cos y = 1$
- 5 Find an equation for the tangent to each curve at the given point on the curve.
- a** $x^2 + y^2 - 3y - 2 = 0$, $(2, 1)$ **b** $2x^2 - xy + y^2 = 28$, $(3, 5)$
c $4 \sin y - \sec x = 0$, $(\frac{\pi}{3}, \frac{\pi}{6})$ **d** $2 \tan x \cos y = 1$, $(\frac{\pi}{4}, \frac{\pi}{3})$
- 6 A curve has the equation $x^2 + 2y^2 - x + 4y = 6$.
- a** Show that $\frac{dy}{dx} = \frac{1-2x}{4(y+1)}$.
b Find an equation for the normal to the curve at the point $(1, -3)$.
- 7 A curve has the equation $x^2 + 4xy - 3y^2 = 36$.
- a** Find an equation for the tangent to the curve at the point $P(4, 2)$.
Given that the tangent to the curve at the point Q on the curve is parallel to the tangent at P ,
b find the coordinates of Q .
- 8 A curve has the equation $y = a^x$, where a is a positive constant.
By first taking logarithms, find an expression for $\frac{dy}{dx}$ in terms of a and x .
- 9 Differentiate with respect to x
- a** 3^x **b** 6^{2x} **c** 5^{1-x} **d** 2^{x^3}
- 10 A biological culture is growing exponentially such that the number of bacteria present, N , at time t minutes is given by
- $$N = 800(1.04)^t.$$
- Find the rate at which the number of bacteria is increasing when there are 4000 bacteria present.